

Inflation Bites Differently: Household Consumption, Welfare, and Poverty during the 2022 European Energy Crisis

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SUPPLEMENTARY MATERIAL

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1 Guide to the Online Supplementary Material.

This Online Supplement provides the technical material supporting the analysis presented in the main paper. The Supplement is intended to ensure the transparency and reproducibility of the quantitative analysis while allowing the main paper to focus on the economic mechanisms and principal findings.

The remainder of the Supplement is organized as follows. Section 2 derives the main theoretical results presented in the paper, including the properties of the non-homothetic CES expenditure function, the household-specific welfare equation, and the comparative-statics results. Section 3 presents the formal existence proof for the general equilibrium together with the proof of the comparative-statics proposition. Section 4 reports additional details on the calibration procedure and compares the observed and fitted expenditure shares. Section 5 provides the complete simulation results for all expenditure groups together with the robustness analysis for alternative values of the elasticity of substitution. Finally, Section 6 describes the calibration of the general-equilibrium extension.

2 Analytical Proofs and Derivations

This section provides the analytical derivations underlying the household demand system, welfare measure, calibration equations, and comparative-static results presented in the main paper. Equations derived locally in this Supplement are numbered (S.1), (S.2), and so forth. References to numbered equations or propositions in the main paper are explicitly identified as such.

Income Effects on Expenditure Shares

This appendix derives the income elasticity of expenditure shares in the nonhomothetic CES framework. Recall that expenditure shares are given by

$$\omega_{g,i} = \frac{\Upsilon_g U_i^{(1-\sigma)\varepsilon_g} p_g^{1-\sigma}}{\sum_{k \in \{E,N,L\}} \Upsilon_k U_i^{(1-\sigma)\varepsilon_k} p_k^{1-\sigma}}. \quad (\text{S.1})$$

Define

$$a_k \equiv \Upsilon_k U_i^{(1-\sigma)\varepsilon_k} p_k^{1-\sigma},$$

so that

$$\omega_{g,i} = \frac{a_g}{\sum_k a_k}.$$

Log differentiation Taking logs,

$$\log \omega_{g,i} = \log a_g - \log \left(\sum_k a_k \right). \quad (\text{S.2})$$

Differentiating with respect to $\log x_i$ yields

$$\frac{\partial \log \omega_{g,i}}{\partial \log x_i} = \frac{\partial \log a_g}{\partial \log x_i} - \frac{\partial}{\partial \log x_i} \log \left(\sum_k a_k \right).$$

A standard identity implies

$$\frac{\partial}{\partial x} \log \left(\sum_k a_k \right) = \sum_k \frac{a_k}{\sum_j a_j} \frac{\partial \log a_k}{\partial x}.$$

Applying this identity with $x = \log x_i$ gives

$$\frac{\partial}{\partial \log x_i} \log \left(\sum_k a_k \right) = \sum_k \omega_{k,i} \frac{\partial \log a_k}{\partial \log x_i}.$$

Income derivatives Since only U_i depends on income,

$$\frac{\partial \log a_k}{\partial \log x_i} = (1 - \sigma) \varepsilon_k \frac{\partial \log U_i}{\partial \log x_i}.$$

Substituting into the previous expression,

$$\frac{\partial}{\partial \log x_i} \log \left(\sum_k a_k \right) = (1 - \sigma) \left(\sum_k \omega_{k,i} \varepsilon_k \right) \frac{\partial \log U_i}{\partial \log x_i}. \quad (\text{S.3})$$

Consider the expenditure-weighted average

$$\bar{\varepsilon}_i = \sum_k \omega_{k,i} \varepsilon_k. \quad (\text{S.4})$$

Using (S.2) and (S.3) yields

$$\frac{\partial \log \omega_{g,i}}{\partial \log x_i} = (1 - \sigma) (\varepsilon_g - \bar{\varepsilon}_i) \frac{\partial \log U_i}{\partial \log x_i}. \quad (\text{S.5})$$

Consider the expenditure function:

$$E(U_i; p) = \left(\sum_{g \in \{E, N, L\}} \Upsilon_g U_i^{(1-\sigma) \varepsilon_g} p_g^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \quad (\text{S.6})$$

Totally differentiating this equation we obtain

$$\frac{\partial \log E(U_i; p)}{\partial \log U_i} = \sum_g \varepsilon_g \frac{\Upsilon_g U_i^{(1-\sigma)\varepsilon_g} p_g^{1-\sigma}}{\sum_j \Upsilon_j U_i^{(1-\sigma)\varepsilon_j} p_j^{1-\sigma}} = \sum_g \varepsilon_g \omega_{g,i} \equiv \bar{\varepsilon}_i.$$

where the second equality is obtained using the expenditure shares equation (S.1). Inverting this relationship and recalling that Recall that $x_i = E(U_i; p)$ gives

$$\frac{\partial \log U_i}{\partial \log x_i} = \frac{1}{\bar{\varepsilon}_i}. \quad (\text{S.7})$$

Combining this equation with (S.5) we obtain

$$\frac{\partial \log \omega_{g,i}}{\partial \log x_i} = (1 - \sigma) \left(\frac{\varepsilon_g}{\bar{\varepsilon}_i} - 1 \right).$$

This completes the derivation of equation (7) in the main paper.

Derivation of the Household-specific Price Index

The exact household-specific cost-of-living index is

$$P_i(p, p^0; U_i^0) = \frac{E(U_i^0; p)}{E(U_i^0; p^0)}. \quad (\text{S.8})$$

Using the NHCES expenditure function,

$$P_i(p, p^0; U_i^0) = \left[\frac{\sum_g \Upsilon_g (U_i^0)^{(1-\sigma)\varepsilon_g} p_g^{1-\sigma}}{\sum_g \Upsilon_g (U_i^0)^{(1-\sigma)\varepsilon_g} (p_g^0)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}}.$$

Since

$$p_g^{1-\sigma} = (p_g^0)^{1-\sigma} \left(\frac{p_g}{p_g^0} \right)^{1-\sigma},$$

we can write

$$P_i(p, p^0; U_i^0) = \left[\sum_g \frac{\Upsilon_g (U_i^0)^{(1-\sigma)\varepsilon_g} (p_g^0)^{1-\sigma}}{\sum_k \Upsilon_k (U_i^0)^{(1-\sigma)\varepsilon_k} (p_k^0)^{1-\sigma}} \left(\frac{p_g}{p_g^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

The fraction multiplying each relative-price term is the base-period expenditure share,

$$\omega_{g,i}^0 = \frac{\Upsilon_g (U_i^0)^{(1-\sigma)\varepsilon_g} (p_g^0)^{1-\sigma}}{\sum_k \Upsilon_k (U_i^0)^{(1-\sigma)\varepsilon_k} (p_k^0)^{1-\sigma}}. \quad (\text{S.9})$$

Therefore,

$$P_i(p, p^0; U_i^0) = \left[\sum_g \omega_{g,i}^0 \left(\frac{p_g}{p_g^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (\text{S.10})$$

Derivation of the marginal utility λ_i

Consider the household's indirect utility:

$$\mathcal{V}_i = \log(U_i) - \Psi_i(n_i), \quad (\text{S.11})$$

We have:

$$\lambda_i \equiv \frac{\partial \mathcal{V}_i}{\partial x_i} = \frac{\partial \log U_i}{\partial x_i}.$$

Using equation (S.7), we easily get:

$$\lambda_i = \frac{\partial \log U_i}{\partial x_i} = \frac{1}{x_i \bar{\varepsilon}_i}. \quad (\text{S.12})$$

Derivation the Household Welfare Equation

This appendix derives the local relationship between nominal expenditure, utility, and the exact household-specific cost-of-living index. We take logs of the expenditure function (S.6):

$$\log x_i = \frac{1}{1-\sigma} \log \left[\sum_{g \in \{E, N, L\}} \Upsilon_g U_i^{(1-\sigma)\varepsilon_g} p_g^{1-\sigma} \right]. \quad (\text{S.13})$$

Define

$$A_{g,i} \equiv \Upsilon_g U_i^{(1-\sigma)\varepsilon_g} p_g^{1-\sigma}$$

Then, from equation (S.1):

$$\omega_{g,i} = \frac{A_{g,i}}{\sum_k A_{k,i}}.$$

Totally differentiating (S.13) gives

$$d \log x_i = \frac{1}{1-\sigma} \sum_g \omega_{g,i} d \log A_{g,i}.$$

Since

$$\log A_{g,i} = \log \Upsilon_g + (1-\sigma)\varepsilon_g \log U_i + (1-\sigma) \log p_g,$$

we have

$$d \log A_{g,i} = (1-\sigma)\varepsilon_g d \log U_i + (1-\sigma) d \log p_g.$$

Substituting into the previous expression yields

$$\begin{aligned} d \log x_i &= \frac{1}{1-\sigma} \sum_g \omega_{g,i} [(1-\sigma)\varepsilon_g d \log U_i + (1-\sigma)d \log p_g] \\ &= \left(\sum_g \omega_{g,i} \varepsilon_g \right) d \log U_i + \sum_g \omega_{g,i} d \log p_g. \end{aligned}$$

Using the definition of $\bar{\varepsilon}_i$ in equation (S.4),

$$d \log x_i = \bar{\varepsilon}_i d \log U_i + \sum_g \omega_{g,i} d \log p_g. \quad (\text{S.14})$$

Now we show that

$$d \log P_i = \sum_g \omega_{g,i} d \log p_g \quad (\text{S.15})$$

Consider first the definition of $P_i(p, p^0; U_i^0)$ in equation (9) in the paper. Since the denominator is evaluated at the base price vector p^0 , it is constant when current prices change. Hence,

$$d \log P_i = d \log E(U_i^0; p). \quad (\text{S.16})$$

By Shephard's lemma,

$$\frac{\partial E(U_i^0; p)}{\partial p_g} = Q_{g,i}^h(U_i^0; p).$$

Therefore,

$$d \log E(U_i^0; p) = \sum_{g \in \{E, N, L\}} \frac{\partial E(U_i^0; p)}{\partial p_g} \frac{p_g}{E(U_i^0; p)} d \log p_g \quad (\text{S.17})$$

$$= \sum_{g \in \{E, N, L\}} \omega_{g,i} d \log p_g, \quad (\text{S.18})$$

since

$$\omega_{g,i} = \frac{p_g Q_{g,i}^h(U_i^0; p)}{E(U_i^0; p)}$$

is the expenditure share of good g evaluated at the reference utility level. Substituting equation (S.15) into equation (S.14) yields

$$d \log x_i = \bar{\varepsilon}_i d \log U_i + d \log P_i,$$

which is equation (11) in the main text.

Derivation of the Calibration Equation

Starting from equations (S.6) and (S.1) and using the identity $x_i = E(U_i; p)$, we obtain

$$\omega_{g,i} = \Upsilon_g U_i^{(1-\sigma)\varepsilon_g} p_g^{1-\sigma} x_i^{\sigma-1}. \quad (\text{S.19})$$

Let L be the base good. Rearranging equation (S.19) for $g = L$ gives

$$U_i^{1-\sigma} = \left(\frac{\omega_{L,i} x_i^{1-\sigma}}{\Upsilon_L p_L^{1-\sigma}} \right)^{1/\varepsilon_L}.$$

Substituting this expression into equation (S.19) for any good $g \neq L$ yields

$$\omega_{g,i} = \Omega_g p_g^{1-\sigma} p_L^{-(1-\sigma)\phi_g} x_i^{(1-\sigma)(\phi_g-1)} \omega_{L,i}^{\phi_g},$$

where $\phi_g \equiv \frac{\varepsilon_g}{\varepsilon_L}$ and $\Omega_g \equiv \frac{\Upsilon_g}{\Upsilon_L^{\phi_g}}$. Taking logs gives

$$\log \omega_{g,i} = \log \Omega_g + (1-\sigma) \log p_g - (1-\sigma)\phi_g \log p_L + (1-\sigma)(\phi_g-1) \log x_i + \phi_g \log \omega_{L,i}.$$

Since prices are common across households in the same year, we define:

$$a_g \equiv \log \Omega_g + (1-\sigma) \log p_g - (1-\sigma)\phi_g \log p_L. \quad (\text{S.20})$$

Therefore, for $g \in \{E, N\}$, we obtain

$$\log \omega_{g,i} = a_g + (1-\sigma)(\phi_g-1) \log x_i + \phi_g \log \omega_{L,i}.$$

This is the equation used in the calibration. This completes the derivation of equation (16) in the main paper.

3 Existence of Equilibrium

All the numbered equations we refer to appear in the main paper.

Definition 1 (General Equilibrium). *Given the exogenous energy price p_E and interest rate r , a static general equilibrium is a collection of allocations*

$$\{Q_{g,i}, n_i\}, \quad \{Y_g, K_g, H_g, E_g\}_{g \in \{N, L\}},$$

and prices (p_N, p_L, w) such that:

1. For each household type i , the vector $(Q_{E,i}, Q_{N,i}, Q_{L,i}, n_i)$ maximizes utility in (1) subject to the budget constraint 4;
2. For each sector $g \in \{N, L\}$, the vector (K_g, H_g, E_g) solves the firm's cost minimization problem 14, and prices satisfy the unit-cost conditions 15;
3. Domestic goods-market identities hold: $Y_g = \sum_i Q_{g,i} + NX_g$, where NX_g denotes net exports of domestic good $g \in \{N, L\}$.
4. The economy satisfies the balanced-trade identity

$$p_E \left(\sum_i Q_{E,i} + E_N + E_L \right) = p_N X_N + p_L X_L.$$

5. The labor market clears, i.e. $\sum_i z_i n_i = H_N + H_L$.

Recall that the exogenous variables of the model are the imported energy price p_E , the interest rate r , sectoral productivity parameters $\{A_g\}_{g \in \{N, L\}}$, production elasticities $\alpha_g, \beta_g, \gamma_g$, and household productivity levels $\{z_i\}_i$. The economy is open. The world energy price p_E and the interest rate r are exogenous, while net exports adjust endogenously to satisfy the external-balance identity. The endogenous variables are:

$$\{Q_{g,i}, n_i, Y_g, K_g, H_g, E_g, p_g, w\}.$$

The key observation is that, conditional on the wage w , all other endogenous variables are pinned down recursively.

Step 1: Prices as functions of the wage

Given the exogenous variables (p_E, r) , firms' cost minimization implies

$$p_g(w) = \frac{1}{A_g} \kappa_g r^{\alpha_g} w^{\beta_g} p_E^{\gamma_g}, \quad g \in \{N, L\},$$

where

$$\kappa_g = \alpha_g^{-\alpha_g} \beta_g^{-\beta_g} \gamma_g^{-\gamma_g}.$$

Hence, for any candidate wage $w > 0$, the vector of goods prices

$$p(w) = (p_E, p_N(w), p_L(w))$$

is uniquely determined and continuous in w .

Step 2: Household optimization

Given the wage w and the associated price vector $p(w)$, each household solves

$$\max_{\{Q_{g,i}\}, n_i} U_i - \Psi_i(n_i)$$

subject to

$$p_E Q_{E,i} + p_N(w) Q_{N,i} + p_L(w) Q_{L,i} = w z_i n_i.$$

Functions U_i implicitly defined in equation (2) in the paper and $\Psi_i(n_i)$ satisfy standard assumptions - continuity, strict monotonicity, and strict quasi-concavity of preferences together with strict convexity of labor disutility - the household problem admits a unique solution. Therefore, for every $w > 0$, household demands and labor supply are well-defined:

$$Q_{g,i}(w), \quad n_i(w).$$

Moreover, these functions are continuous in w . Aggregate labor supply is therefore

$$H^s(w) = \sum_i z_i n_i(w).$$

Step 3: Domestic production and net exports

For every candidate wage, household demands determine domestic absorption of goods N and L . Sectoral outputs are then determined recursively through the production side of the model. Net exports are obtained residually from

$$NX_g(w) = Y_g(w) - \sum_i Q_{g,i}(w), \quad g \in \{N, L\}.$$

Energy imports are equal to $\sum_i Q_{E,i}(w) + E_N(w) + E_L(w)$. Since the economy is open and small compared to the world economy, energy imports and net exports adjust endogenously to satisfy the external-balance identity, while the world price of energy remains exogenous. Consequently, sectoral outputs and net exports are continuous functions of the wage.

Step 4: Factor demands

Given outputs and prices, firms' optimality conditions imply

$$wH_g(w) = \beta_g p_g(w)Y_g(w),$$

$$rK_g(w) = \alpha_g p_g(w)Y_g(w),$$

$$p_E E_g(w) = \gamma_g p_g(w)Y_g(w).$$

Thus,

$$H_g(w) = \frac{\beta_g p_g(w)Y_g(w)}{w},$$

$$K_g(w) = \frac{\alpha_g p_g(w)Y_g(w)}{r},$$

$$E_g(w) = \frac{\gamma_g p_g(w)Y_g(w)}{p_E}.$$

Aggregate labor demand is therefore

$$H^d(w) = H_N(w) + H_L(w).$$

Since prices and outputs are continuous in w , aggregate labor demand is also continuous in w .

Step 5: Excess labor demand

Define excess labor demand as $Z(w) = H^d(w) - H^s(w)$. A competitive equilibrium is a wage w^* such that $Z(w^*) = 0$. Once w^* is determined, all remaining endogenous variables are uniquely pinned down by the previous equations.

Step 6: Existence

Since both labor demand and labor supply are continuous in w , the excess demand function $Z(w)$ is continuous. Suppose there exist two wages w_L and w_H such that $Z(w_L) > 0$ and $Z(w_H) < 0$. By the Intermediate Value Theorem, there exists at least one wage $w^* \in (w_L, w_H)$ such that $Z(w^*) = 0$. Therefore, at least one general equilibrium exists.

Proof of Proposition 1

Consider the unit-cost equation:

$$p_g = \frac{\kappa_g}{A_g} r^{\alpha_g} w^{\beta_g} p_E^{\gamma_g}, \quad g \in \{N, L\}, \quad (\text{S.21})$$

where $\kappa_g = \alpha_g^{-\alpha_g} \beta_g^{-\beta_g} \gamma_g^{-\gamma_g}$. Differentiating (S.21) with respect to $\log p_E$ yields

$$\frac{d \log p_g}{d \log p_E} = \beta_g \frac{d \log w}{d \log p_E} + \gamma_g, \quad g \in \{N, L\}.$$

Let $\eta_w \equiv \frac{d \log w}{d \log p_E}$ denote the elasticity of the equilibrium wage with respect to the energy price. Then

$$\frac{d \log p_N}{d \log p_E} = \beta_N \eta_w + \gamma_N, \quad \frac{d \log p_L}{d \log p_E} = \beta_L \eta_w + \gamma_L. \quad (\text{S.22})$$

Substituting these expressions into the differential representation of the exact cost-of-living index derived in equation (S.15) immediately gives

$$\begin{aligned} \frac{d \log P_i}{d \log p_E} &= \omega_{E,i} + \omega_{N,i}(\gamma_N + \beta_N \eta_w) + \omega_{L,i}(\gamma_L + \beta_L \eta_w) \\ &= \omega_{E,i} + \gamma_N \omega_{N,i} + \gamma_L \omega_{L,i} + \eta_w (\beta_N \omega_{N,i} + \beta_L \omega_{L,i}). \end{aligned}$$

4 Calibration

4.1 COICOP Categories and Consumption Baskets

To analyze the distributional impact of the 2022 price shock, we aggregate household expenditures into three mutually exclusive consumption baskets: Energy (E), Luxuries (L), and Necessities (N). Table 1 details the mapping between these analytical baskets and the standard COICOP (Classification of Individual Consumption According to Purpose) divisions used in both the Spanish and Italian household budget surveys.

We adopt a narrow definition of energy for our baseline descriptive statistics, restricting the Energy basket strictly to domestic electricity, gas, and other heating fuels (COICOP 04.5). The Luxuries basket comprises non-essential goods and services, including furnishings, communication, recreation, restaurants, and miscellaneous services (COICOP 05, 08, 09, 11, and 12).

Finally, the Necessities basket is derived as a residual category ($N = \text{Total Expenditure} - E - L$). As a result, it encompasses all basic subsistence expenses such as food, clothing, and health, as well as broader household needs like water supply, dwelling services, and transport fuels, which are excluded from our narrow baseline energy definition.

Table 1: Composition of Consumption Baskets by COICOP Categories (Spain and Italy)

Consumption Basket	COICOP Divisions and Classes
Luxuries (L)	05 (Furnishings, household equipment and routine household maintenance), 08 (Communication), 09 (Recreation and culture), 11 (Restaurants and hotels), 12 (Miscellaneous goods and services)
Energy (E)	04.5 (Electricity, gas and other fuels)
Necessities (N)	Calculated as residual ($N = \text{Total Expenditure} - E - L$). Primarily includes: 01 (Food and non-alcoholic beverages), 02 (Alcoholic beverages, tobacco and narcotics), 03 (Clothing and footwear), 06 (Health), 07 (Transport), 10 (Education), plus non-energy components of division 04 (e.g., rentals, water supply, dwelling services).

4.2 Results of the Minimum Distance Procedure

"Vent"	"E*obs"	"E*fit"	"E*err"	"N*obs"	"fit*N"	"err*N"	"L*obs"	"fit*L"	"err*L"
1	0.0741	0.0682	0.0059	0.7297	0.7357	-0.0060	0.1962	0.1961	0.0000
2	0.0662	0.0608	0.0054	0.7258	0.7264	-0.0005	0.2080	0.2128	-0.0048
3	0.0646	0.0571	0.0075	0.7097	0.7206	-0.0109	0.2257	0.2223	0.0034
4	0.0609	0.0546	0.0063	0.7184	0.7163	0.0021	0.2207	0.2291	-0.0084
5	0.0587	0.0530	0.0057	0.7135	0.7133	0.0002	0.2278	0.2338	-0.0059
6	0.0551	0.0509	0.0042	0.7103	0.7093	0.0010	0.2346	0.2398	-0.0051
7	0.0514	0.0501	0.0014	0.7134	0.7075	0.0059	0.2352	0.2424	-0.0072
8	0.0520	0.0489	0.0030	0.7035	0.7051	-0.0016	0.2445	0.2460	-0.0014
9	0.0494	0.0475	0.0018	0.6981	0.7019	-0.0038	0.2525	0.2506	0.0019
10	0.0472	0.0464	0.0007	0.6991	0.6993	-0.0003	0.2538	0.2542	-0.0005
11	0.0465	0.0454	0.0012	0.6898	0.6966	-0.0069	0.2637	0.2580	0.0057
12	0.0439	0.0438	0.0001	0.6850	0.6925	-0.0076	0.2712	0.2637	0.0074
13	0.0416	0.0430	-0.0014	0.6892	0.6906	-0.0014	0.2692	0.2663	0.0028
14	0.0418	0.0419	-0.0001	0.6953	0.6874	0.0079	0.2630	0.2707	-0.0077
15	0.0375	0.0401	-0.0026	0.6786	0.6820	-0.0034	0.2840	0.2780	0.0060
16	0.0369	0.0393	-0.0024	0.6746	0.6798	-0.0052	0.2885	0.2809	0.0077
17	0.0332	0.0380	-0.0048	0.6677	0.6755	-0.0077	0.2991	0.2865	0.0126
18	0.0319	0.0362	-0.0043	0.6779	0.6692	0.0087	0.2902	0.2947	-0.0044
19	0.0302	0.0342	-0.0039	0.6560	0.6618	-0.0058	0.3137	0.3040	0.0097
20	0.0245	0.0299	-0.0054	0.6654	0.6439	0.0215	0.3100	0.3262	-0.0161

Table 2: Observed and fitted expenditure shares by income ventile: Spain.

"Vent"	"E*obs"	"E*fit"	"E*err"	"N*obs"	"fit*N"	"err*N"	"L*obs"	"fit*L"	"err*L"
1	0.0976	0.0849	0.0128	0.7827	0.7994	-0.0167	0.1197	0.1157	0.0039
2	0.0871	0.0751	0.0120	0.7839	0.7885	-0.0046	0.1290	0.1364	-0.0074
3	0.0786	0.0710	0.0076	0.7818	0.7825	-0.0007	0.1397	0.1465	-0.0068
4	0.0772	0.0689	0.0083	0.7777	0.7791	-0.0013	0.1451	0.1520	-0.0069
5	0.0754	0.0659	0.0095	0.7728	0.7736	-0.0008	0.1517	0.1604	-0.0087
6	0.0704	0.0638	0.0066	0.7680	0.7694	-0.0014	0.1616	0.1668	-0.0052
7	0.0690	0.0627	0.0063	0.7690	0.7671	0.0020	0.1619	0.1702	-0.0083
8	0.0640	0.0607	0.0033	0.7661	0.7625	0.0036	0.1700	0.1768	-0.0069
9	0.0622	0.0586	0.0036	0.7486	0.7574	-0.0088	0.1891	0.1840	0.0051
10	0.0600	0.0571	0.0029	0.7535	0.7535	0.0000	0.1865	0.1894	-0.0029
11	0.0576	0.0557	0.0019	0.7450	0.7498	-0.0048	0.1974	0.1944	0.0029
12	0.0547	0.0542	0.0006	0.7357	0.7453	-0.0096	0.2096	0.2005	0.0090
13	0.0522	0.0527	-0.0005	0.7320	0.7408	-0.0088	0.2158	0.2065	0.0093
14	0.0464	0.0512	-0.0048	0.7393	0.7362	0.0031	0.2143	0.2126	0.0017
15	0.0461	0.0498	-0.0038	0.7214	0.7315	-0.0101	0.2326	0.2187	0.0139
16	0.0447	0.0479	-0.0032	0.7216	0.7245	-0.0029	0.2337	0.2276	0.0061
17	0.0418	0.0465	-0.0047	0.7201	0.7192	0.0009	0.2381	0.2343	0.0038
18	0.0390	0.0443	-0.0052	0.7132	0.7101	0.0031	0.2477	0.2456	0.0021
19	0.0332	0.0416	-0.0084	0.7018	0.6981	0.0037	0.2650	0.2603	0.0047
20	0.0256	0.0356	-0.0100	0.6918	0.6673	0.0246	0.2825	0.2971	-0.0146

Table 3: Observed and fitted expenditure shares by income ventile: Italy.

4.3 Recovery of the preference weights

The minimum-distance procedure presented in section 3.2 in the main paper identifies the transformed parameters a_E and a_N . defined as

$$a_g = \log \frac{\Upsilon_g}{\Upsilon_L} + (1 - \sigma) \log p_g - (1 - \sigma) \phi_g \log p_L. \quad (\text{S.23})$$

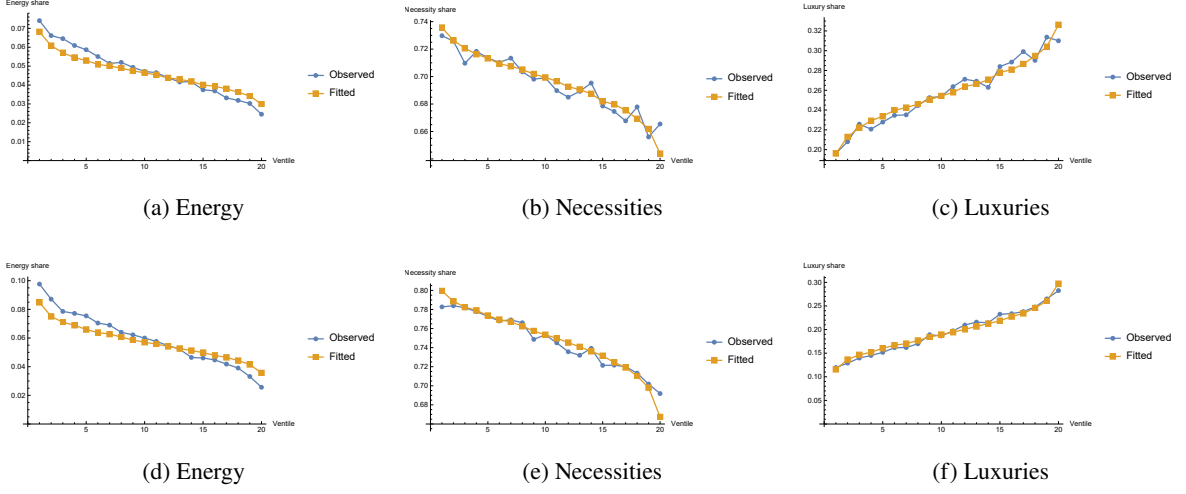


Figure 1: Observed and fitted expenditure shares by expenditure ventile. Panels (a)–(c) report the results for Spain, while panels (d)–(f) report the results for Italy.

We get $a_E = 1.5$ and $a_N = 3.1$ for Italy, and $a_E = 1.3$ and $a_N = 2.7$ for Spain. To recover the preference weights Υ_g for $g \in \{E, N, L\}$ we use the normalization adopted by [Comin et al. \(2021\)](#). Define $\Omega_g = \frac{\Upsilon_g}{\Upsilon_L^{\phi_g}}$. We normalize $\Upsilon_L = 1$, so that $\Omega_g = \Upsilon_g$. Moreover, because benchmark prices are normalized to unity, $p_E = p_N = p_L = 1$, equation (S.23) implies $\Upsilon_g = e^{a_g}$, for $g \in \{E, N\}$. These parameters ensure that the model exactly reproduces the average expenditure shares implied by the estimated NHCES demand system. They have no independent economic interpretation because they depend on the normalization of both utility and prices.

4.4 Utility elasticity of expenditure across expenditure ventiles

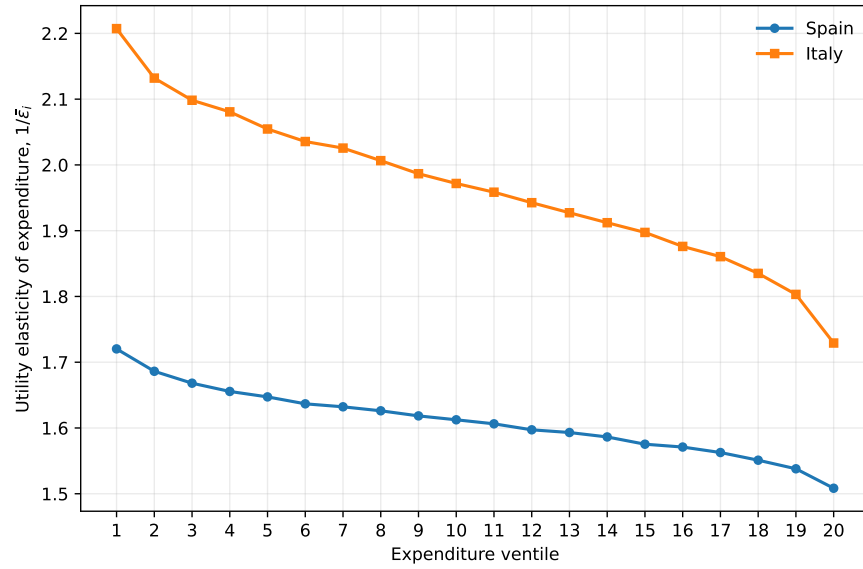


Figure 2: **Utility elasticity of expenditure across expenditure ventiles.** The figure reports $\frac{\partial \log U_i}{\partial \log x_i} = \frac{1}{\varepsilon_i}$, for each expenditure ventile in Spain and Italy.

5 Additional Simulations Results

Expenditure Ventile	Inflation (%)	Change in log utility ($\times 100$)
1	8.08	-13.51
2	7.87	-12.91
3	7.77	-12.59
4	7.69	-12.38
5	7.65	-12.25
6	7.59	-12.07
7	7.56	-12.00
8	7.53	-11.90
9	7.48	-11.78
10	7.45	-11.68
11	7.42	-11.59
12	7.37	-11.45
13	7.35	-11.38
14	7.31	-11.28
15	7.25	-11.11
16	7.23	-11.05
17	7.19	-10.93
18	7.13	-10.75
19	7.06	-10.57
20	6.92	-10.15

Table 4: Household-specific inflation and utility changes by expenditure ventile in Spain.

Expenditure Ventile	Inflation (%)	Change in log utility ($\times 100$)
1	9.87	-22.85
2	9.15	-20.41
3	8.85	-19.40
4	8.69	-18.89
5	8.47	-18.16
6	8.32	-17.64
7	8.24	-17.38
8	8.08	-16.88
9	7.93	-16.38
10	7.82	-16.01
11	7.72	-15.69
12	7.60	-15.31
13	7.48	-14.96
14	7.38	-14.62
15	7.27	-14.29
16	7.12	-13.83
17	7.02	-13.50
18	6.85	-12.97
19	6.64	-12.34
20	6.18	-10.97

Table 5: Household-specific inflation and utility changes by expenditure ventile in Italy.

5.1 Sensitivity on the elasticity of substitution

Country	σ	π V1	π V20	Welfare V1	Welfare V20	Δ poverty rate	Heterogeneous π
Spain	0.30	8.43	6.81	-12.24	-8.83	6.34	3.34
	0.56	8.14	6.96	-14.84	-10.84	6.34	3.34
	0.80	7.72	7.09	-18.18	-13.76	3.00	0.00
Italy	0.30	12.02	5.76	-23.25	-8.74	6.19	3.51
	0.56	9.84	6.15	-20.68	-10.28	6.19	3.22
	0.80	8.34	6.69	-30.83	-16.39	6.19	0.00

Table 6: Sensitivity to the elasticity of substitution. Inflation π and welfare are reported for the poorest (V1) and richest (V20) expenditure ventiles. The last two columns report the increase in the poverty rate (percentage points) and the share attributable to heterogeneous household-specific inflation.

As a robustness check, we repeat the quantitative analysis of the baseline model presented in section 3.3 for alternative values of the elasticity of substitution. We set $\sigma = 0.3$ and $\sigma = 0.8$ instead of the benchmark value $\sigma = 0.56$. Table 6 reports the main results for Spain and Italy.

The qualitative conclusions of the paper are robust to alternative assumptions on substitution possibilities. In both countries, the energy shock generates higher inflation, lower real expenditure, and larger welfare losses for poorer households under all parameterizations. Moreover, the ranking of households is unchanged: the poorest ventiles always experience the highest inflation and the largest welfare losses.

The magnitude of these effects, however, differs across outcomes. Household-specific inflation is only mildly affected by the choice of σ . As substitution becomes easier, inflation differences across households narrow because consumers can more readily adjust their expenditure baskets in response to relative-price changes. Nevertheless, the inflation ranking remains virtually unchanged.

By contrast, welfare losses are considerably more sensitive to the elasticity of substitution. Changing σ affects welfare through two distinct channels. First, it alters households' ability to substitute across goods following the energy-price shock. Second, because the preference parameters ε_g for each value of σ , it also changes the average non-homotheticity parameter $\bar{\varepsilon}_i = \sum_g \omega_{ig} \varepsilon_g$, which governs the relationship between inflation and utility (see the welfare equation (11) in the main paper). The

calibration results show that $\bar{\varepsilon}_i$ changes substantially across alternative values of σ , although not necessarily in the same direction across countries. Consequently, variations in σ affect welfare both through households' substitution responses and through the recalibrated non-homothetic demand system. Welfare losses are therefore considerably more sensitive to the assumed elasticity of substitution than household-specific inflation.

Finally, the role of heterogeneous inflation in explaining poverty proves to be highly sensitive to the elasticity of substitution. When substitution possibilities are limited ($\sigma = 0.3$), household-specific inflation makes a substantial additional contribution to the increase in poverty. By contrast, when substitution is sufficiently easy ($\sigma = 0.8$), inflation rates become much more similar across households and the additional contribution of heterogeneous inflation to poverty becomes negligible in both Spain and Italy.

Overall, the sensitivity analysis confirms that the central message of the paper is robust. Regardless of the value of σ , energy-price shocks generate substantial inflation inequality and disproportionately large welfare losses for poorer households, although the quantitative importance of heterogeneous inflation for poverty depends critically on households' ability to substitute across goods.

6 Calibration of the General Equilibrium Extension

The general equilibrium extension preserves the household side of the benchmark model presented in Section 3.2 of the main paper. Consequently, the parameters governing household preferences (namely, the NHCES expenditure-function parameters $\Upsilon_E, \Upsilon_N, \Upsilon_L, \varepsilon_E, \varepsilon_N, \varepsilon_L, \sigma$) are identical to those employed in the benchmark calibration. As a result, household-specific expenditure shares, expenditure elasticities and exact cost-of-living indices are unchanged. The additional calibration concerns only the production side of the economy and the labour market. Table 7 summarizes the additional parameters introduced by the general equilibrium extension together with their empirical sources.

Recall from section 2 in the main paper that the production side consists of two domestic sectors, producing necessity and luxury goods using Cobb–Douglas technologies with capital, labour and imported energy. The labour and energy cost shares are calibrated using official [Eurostat \(2024a\)](#) and [Eurostat \(2024b\)](#) National Accounts. Following the accounting identity for gross output, we

Parameter	Spain	Italy	Source
Labour cost share ($\beta_N = \beta_L$)	0.25	0.20	Eurostat (2024b)
Energy cost share ($\gamma_N = \gamma_L$)	0.03	0.03	Eurostat (2024a)
Inverse Frisch elasticity (φ)	1	1	Chetty et al. (2011)
Sectoral output ratio (V_N/V_L)	2.81	3.39	Eurostat (Eurostat, 2024c)
Residual price shifter (Ξ_N)	1.07	1.04	Authors' calibration
Residual price shifter (Ξ_L)	1.05	1.02	Authors' calibration
Household labour-income profile ($\theta_{W,i}$)	20 ventiles	20 ventiles	Household Surveys

Table 7: Additional calibration parameters of the general equilibrium extension. Household surveys: [Instituto Nacional de Estadística \(2023\)](#) and [Bank of Italy \(2024\)](#).

measure the energy cost share as the ratio between firms' expenditure on energy products and total gross output (variable $P1$). Energy expenditures are obtained from the [Eurostat \(2024b\)](#) Input–Output Tables, while gross output is taken from the National Accounts. This yields benchmark energy cost shares of approximately 3% for both Spain and Italy immediately before the energy crisis.¹ The labour cost share is computed using Eurostat National Accounts (dataset [Eurostat \(2024a\)](#)) as the ratio between compensation of employees (variable $D1$) and gross output (variable $P1$). This measure is conceptually consistent with the Cobb–Douglas production function, since both labour compensation and energy expenditures are expressed relative to total output. The resulting benchmark labour shares are approximately 25% for Spain and 20% for Italy.

Unlike the benchmark model, the production block requires benchmark values of sectoral gross output. These are identified using two restrictions. The first is the observed Eurostat ratio between the values of gross output in the necessity and luxury sectors,

$$\eta_V = \frac{p_N Y_N}{p_L Y_L}. \quad (\text{S.24})$$

¹The [Eurostat \(2024b\)](#) Supply and Use Tables report energy expenditure relative to intermediate consumption rather than gross output. Since intermediate consumption represents only a fraction of total production costs, these ratios are substantially larger and are therefore not directly comparable with the Cobb–Douglas production shares used in our model. We therefore calibrate γ_g using energy expenditure relative to gross output.

The second restriction is provided by the benchmark external-balance identity, which implies

$$(1 - \gamma_N)p_N Y_N + (1 - \gamma_L)p_L Y_L = \sum_i x_i.$$

The first equation determines the relative size of the two sectors, while the second pins down their aggregate scale. Together they uniquely determine benchmark gross output:

$$p_L Y_L = \frac{\sum_i x_i}{(1 - \gamma_N)\eta_V + (1 - \gamma_L)}, \quad p_N Y_N = \eta_V p_L Y_L.$$

Benchmark output quantities follow immediately from

$$Y_g = \frac{p_g Y_g}{p_g},$$

while benchmark net exports are recovered residually from the goods-market identities.

The labour-market extension requires calibrating the composition of household income. Unlike the benchmark model, where household expenditure is taken as exogenous, the general equilibrium model determines household income endogenously through factor payments. Specifically, households receive labour income $w(H_N + H_L)$ by supplying labour to domestic firms and capital income $r(K_N + K_L)$ through their ownership of the aggregate capital stock. Since labour supply responds endogenously to changes in wages, the relative importance of labour and capital income becomes a key determinant of the distributional consequences of the energy shock.

For this reason, the model requires information on the fraction of household income that originates from labour earnings. We obtain this information from nationally representative household surveys². In both countries households are ranked according to equivalised disposable income using the official equivalence scale and then divided into population ventiles taking sampling weights and household size into account. For every ventile we compute the observed labour-income share,

$$\theta_{w,i} = \frac{\text{labour income}_i}{\text{equivalised disposable household income}_i}.$$

Since aggregate labour income obtained from the household surveys differs slightly from aggregate labour compensation implied by the production side, the observed labour-income profile is rescaled

²For Spain we use [Instituto Nacional de Estadística \(2023\)](#), referring to household incomes received in 2022. For Italy we use [Bank of Italy \(2024\)](#).

proportionally so that aggregate household labour income exactly matches labour compensation in the benchmark equilibrium³.

Residual household income is interpreted as non-labour income and determines household-specific ownership shares of aggregate capital income.

Pre-shock labour supply is normalized to one for every household, $n_i = 1$, for $i = 1, \dots, 20$. Consequently, from equation (3) in the paper, household-specific labour endowments are recovered directly from observed labour income according to

$$z_i = \frac{\theta_{W,i} x_i}{w n_i}.$$

Since wages are normalized to unity, this simply implies $z_i = \theta_{W,i} x_i$.

Labour disutility is assumed to take the isoelastic form $\Psi_i(n_i) = \frac{\chi_i}{1+\varphi} n_i^{1+\varphi}$, where φ denotes the inverse Frisch elasticity. Following the empirical evidence summarized by [Chetty \(2012\)](#) and the benchmark calibrations in quantitative macroeconomic models (for instance, [Heathcote et al. \(2017\)](#)), we set $\varphi = 1$, corresponding to a Frisch elasticity equal to one.

Recall that the first-order condition for labor supply is

$$\Psi'_i(n_i) = w z_i \lambda_i, \tag{S.25}$$

where $\lambda_i = \partial \mathcal{V}_i / \partial x_i$ is the marginal utility of income. Evaluating equation (S.25) at the equilibrium, where $n_i = 1$, and using equation (S.12) gives

$$\chi_i = w z_i \lambda_i = \frac{w z_i}{\bar{\epsilon}_i x_i}.$$

Finally, the parsimonious production structure only captures the direct effect of imported energy prices on firms' production costs. To reproduce the observed producer-price inflation experienced during the energy crisis, we introduce sector-specific multiplicative price shifters, Ξ_N and Ξ_L . These residual shifters summarize inflationary forces not explicitly modelled by the production block—including supply-chain disruptions, imported intermediate inputs other than energy, and other sector-specific cost pressures. They are calibrated so that, holding benchmark factor prices fixed, the model exactly reproduces the observed producer-price increases in both sectors.

³The detailed procedure is available on request.

The model is closed as a small open economy. The world rental rate of capital is taken as exogenous, while wages adjust endogenously to clear the labour market. Benchmark exports are kept fixed at their observed levels, whereas international capital flows adjust endogenously to accommodate changes in domestic capital demand. Any remaining current-account imbalance is absorbed through foreign borrowing, which is zero in the benchmark equilibrium.

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